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KALAFUT, STEPHEN J	

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Please find below and/or attached an Office communication concerning this application or proceeding.

The time period for reply, if any, is set in the attached communication.

Office Action Summary	**Application No.** 10/552,585	**Applicant(s)** MILLS, RANDELL L.
	Examiner Stephen J. Kalafut	**Art Unit** 1795

-- The MAILING DATE of this communication appears on the cover sheet with the correspondence address --

Period for Reply

A SHORTENED STATUTORY PERIOD FOR REPLY IS SET TO EXPIRE *3* MONTH(S) OR THIRTY (30) DAYS, WHICHEVER IS LONGER, FROM THE MAILING DATE OF THIS COMMUNICATION.

- Extensions of time may be available under the provisions of 37 CFR 1.136(a). In no event, however, may a reply be timely filed after SIX (6) MONTHS from the mailing date of this communication.
- If NO period for reply is specified above, the maximum statutory period will apply and will expire SIX (6) MONTHS from the mailing date of this communication.
- Failure to reply within the set or extended period for reply will, by statute, cause the application to become ABANDONED (35 U.S.C. § 133). Any reply received by the Office later than three months after the mailing date of this communication, even if timely filed, may reduce any earned patent term adjustment. See 37 CFR 1.704(b).

Status

1)☐ Responsive to communication(s) filed on _____.

2a)☐ This action is **FINAL**. 2b)☒ This action is non-final.

3)☐ Since this application is in condition for allowance except for formal matters, prosecution as to the merits is closed in accordance with the practice under *Ex parte Quayle*, 1935 C.D. 11, 453 O.G. 213.

Disposition of Claims

4)☒ Claim(s) *1-316* is/are pending in the application.

4a) Of the above claim(s) _____ is/are withdrawn from consideration.

5)☐ Claim(s) _____ is/are allowed.

6)☒ Claim(s) *1-316* is/are rejected.

7)☐ Claim(s) _____ is/are objected to.

8)☐ Claim(s) _____ are subject to restriction and/or election requirement.

Application Papers

9)☐ The specification is objected to by the Examiner.

10)☒ The drawing(s) filed on *12 October 2005* is/are: a)☒ accepted or b)☐ objected to by the Examiner.

Applicant may not request that any objection to the drawing(s) be held in abeyance. See 37 CFR 1.85(a).

Replacement drawing sheet(s) including the correction is required if the drawing(s) is objected to. See 37 CFR 1.121(d).

11)☐ The oath or declaration is objected to by the Examiner. Note the attached Office Action or form PTO-152.

Priority under 35 U.S.C. § 119

12)☐ Acknowledgment is made of a claim for foreign priority under 35 U.S.C. § 119(a)-(d) or (f).

a)☐ All b)☐ Some * c)☐ None of:

1.☐ Certified copies of the priority documents have been received.

2.☐ Certified copies of the priority documents have been received in Application No. _____.

3.☐ Copies of the certified copies of the priority documents have been received in this National Stage application from the International Bureau (PCT Rule 17.2(a)).

* See the attached detailed Office action for a list of the certified copies not received.

Attachment(s)

1) ☒ Notice of References Cited (PTO-892)

2) ☐ Notice of Draftsperson's Patent Drawing Review (PTO-948)

3) ☒ Information Disclosure Statement(s) (PTO/SB/08) Paper No(s)/Mail Date *13 Feb 2006*.

4) ☐ Interview Summary (PTO-413) Paper No(s)/Mail Date. _____ .

5) ☐ Notice of Informal Patent Application

6) ☐ Other: _____.

35 U.S.C. 101 reads as follows:

> Whoever invents or discovers any new and useful process, machine, manufacture, or composition of matter, or any new and useful improvement thereof, may obtain a patent therefor, subject to the conditions and requirements of this title.

Claims 1-316 are rejected under 35 U.S.C. 101 because the disclosed invention is inoperative and therefore lacks credibly utility. All the claims recite a reactor in which a plasma includes a "lower-energy hydrogen", or a method of producing power and such a hydrogen species using such a reactor. Such a hydrogen species is presently defined as a hydrogen atom having the binding energy given in the following equation, which recites: Binding Energy = 13.6 eV/n^2, where n = l/p and p is an integer greater than 1. The present claims encompass compounds with such hydrogen atoms which have been ionized, either positively or negatively, or are neutral but which in each case exhibit the increased binding energy. The specification, such as on pages 11 and 12, uses the term "hydrino" to describe these hydrogen species. All of the species presently contemplated, however, are based on applicant's assertion that hydrogen may have energy states that are below the conventionally accepted ground state, as expressed by the equation above.

An asserted utility would not be considered credible where a person of ordinary skill would consider the assertion to be incredible in view of contemporary knowledge and where the evidence offered by an applicant does not counter what contemporary knowledge otherwise suggests. According to conventionally accepted scientific principle, the existence of hydrogen with a binding energy corresponding to a value of "n" which is not an integer cannot be mathematically justified. See the attached *Appendix*. According to "Endnote 1" of the *Appendix*, Schrödinger's wave equation mandates that the value of "n" must be a positive integer

(1, 2, 3, etc.). According to "Endnote V", fractional values for "n" are also impossible in light of the Uncertainty Principle. The fourth full paragraph of page 19-14 of Bethe and Salpeter's *Quantum Mechanics of One- and Two- Electron Atoms* (Plenum Publishing Corporation, New York, 1977), cited on page 13 of the IDS of 30 January 2006, states that the "ground state" of hydrogen has n = 1. Since applicant's invention is based on a form of hydrogen, which according to conventionally accepted scientific principle cannot exist, the invention would be inoperative and thus lack utility. Thus burden is shifted to applicant to provide satisfactory evidence of the operability of the invention, Newman v. Quigg, 877 F2d 1575, 11 USPQ2d 1340 (Fd. Cr. 1989).

The following is a quotation of the first paragraph of 35 U.S.C. 112:

> The specification shall contain a written description of the invention, and of the manner and process of making and using it, in such full, clear, concise, and exact terms as to enable any person skilled in the art to which it pertains, or with which it is most nearly connected, to make and use the same and shall set forth the best mode contemplated by the inventor of carrying out his invention.

Claims 1-316 are rejected under 35 U.S.C. 112, first paragraph, as failing to comply with the enablement requirement. The claim(s) contains subject matter which was not described in the specification in such a way as to enable one skilled in the art to which it pertains, or with which it is most nearly connected, to make and/or use the invention. The specification does not enable one of ordinary skill in the art to make or use compounds including a hydrogen species having an "increased binding energy", in that undue experimentation would be required.

Factors to be considered in determining whether a disclosure would require undue experimentation, as set forth by *in re Wands*, 8 USPQ2d 1400, 1404 (Fed. Cir. 1988), include:

(1) The quantity of experimentation necessary

(2) The amount of direction of guidance presented in the specification,

(3) The presence or absence of working examples,

(4) The nature of the invention,

(5) The state of the prior art,

(6) The relative skill of those in the art

(7) The predictability or unpredictability of the art

(8) The breadth of the claims

Each of these factors will be addressed as to their relevance to the lack of enablement of the present claims.

<u>(1) The Quantity of Experimentation Necessary</u>

Pages 19 through 21 of the present specification refer to hydrogen with an increased binding energy being prepared in by means of plasma in a "plasma electrolysis cell hydride reactor". Pages 24 through 26 refer to a "plasma torch cell reactor". Pages 28 and 29 refer to a "plasma gas cell hydride and power reactor". Plasma is a heated ionized gas in which electrons are removed from their corresponding atomic nuclei, which would be the opposite of increasing the energy of their bond to the nuclei. In all of these plasma-dependent embodiments, the artisan would thus face the initial challenge of getting the electrons of hydrogen atoms to move closer to the nuclei in an environment which would tend to separate the electrons and nuclei.

<u>(2) The Amount of Direction or Guidance Presented in the Specification</u>

On pages 29 through 32, the specification gives some general guidelines as to the voltage and frequency of the alternating current power input, and the partial pressure of the reactants, and the amount of microwave power input.

(3) The Presence or Absence of Working Examples

The specification does not contain any specific examples of any of the present plasma or gas reactors being used, along with any specific values for voltage or frequency of the power input, or the partial pressure of the reactants.

(4) The Nature of the Invention

The scientific community has held the belief for decades that hydrogen cannot exist below the "ground state" where n = 1. See the rejection above under §101. The nature of the invention is that it is based on forms of hydrogen which cannot exist under the accepted laws of physics and mathematics. Thus, in order to establish enablement, applicant bears the burden of proving the accepted scientific laws wrong or incomplete, which is not done in the present specification and the examples on pages 38-43. Applicant himself recognizes the unusual nature of his invention. In his book The *Grand Unified Theory of Classical Quantum Mechanics*, on page 14, applicant states that the theory underlying his invention "predicts the existence of a previously unknown form of matter: hydrogen atoms and molecules having electrons of lower energy than the conventional 'ground' state".

(5) The State of the Prior Art

There appears to be no prior art, other than by applicant, showing hydrogen with a binding energy corresponding to "n" being a fraction, below the integer 1, or even any prior art which suggest that this hydrogen could exist in theory. There is thus no prior art, in light of which applicant's disclosure may be read, which would help to enable the existence of compounds having the "lower-energy" species of hydrogen.

(6) Relative Skill of Those in the Art

The most highly skilled people in the art, physicists and chemists familiar with quantum mechanics, would regard the present hydrogen species as something that cannot exist, for reasons set forth in the attached *Appendix*. See also the article by Krieg, "Hydrino: A State below the ground state", cited on page 40 of the IDS.

(7) The Predictability or Unpredictability of the Art

Because the hydrogen atom with a binding energy corresponding to "n" being less than 1 would be regarded as something that cannot exist, and because the present types of hydrogen species are based on "n" being less than 1, the recognized state of the art would predict against the present compounds which include these forms of hydrogen from ever being formed. At best, since the state of the art does not recognize hydrogen species with a "lower energy", predicting how any given hydrogen species or compound within the present claims is formed would be extremely difficult, even if these hydrogen species were shown to exist.

(8) The Breadth of the Claims

The present claims encompass reactors using plasmas to form hydrogen of "lower-energy", or process of using such a reactor for to make power and such hydrogen, more broadly than shown by the general guidelines in the specification, and without any specific examples. The specification also does not show how applicant's process can be controlled to product the different levels of "p" (where $n = 1/p$) which are encompassed by the present claims.

In conclusion, the present disclose would require undue experimentation, as seen from a consideration of the above factors. This is mainly because factor (1), the amount of required experimentation, and factor (8), the breadth of the claims, are greater than factors (2), the amount of guidance, which for reasons above, would tend to lead the artisan away from a hydrogen atom

with its electron closer to its nucleus than normal, and (3), the nature of the example, which does not show the artisan how to obtain a desired value for "p". For this reason, and because factors (4) through (7) would show that the invention is based on a type of hydrogen species which cannot exist, the present disclosure is considered to be non-enabling.

The prior art made of record and not relied upon is considered pertinent to applicant's disclosure. Weiler *et al.* (US 6936144) disclose a reactor that forms plasma and uses a magnetic field. Mutterer *et al.* (US 6,258,329) disclose a plasma reactor that uses microwave radiation.

Any inquiry concerning this communication or earlier communications from the examiner should be directed to Stephen J. Kalafut whose telephone number is 571-272-1286. The examiner can normally be reached on Mon-Fri 8:00 am-4:30 pm.

If attempts to reach the examiner by telephone are unsuccessful, the examiner's supervisor, Patrick J. Ryan can be reached on 571-272-1292. The fax phone number for the organization where this application or proceeding is assigned is 571-273-8300.

Information regarding the status of an application may be obtained from the Patent Application Information Retrieval (PAIR) system. Status information for published applications may be obtained from either Private PAIR or Public PAIR. Status information for unpublished applications is available through Private PAIR only. For more information about the PAIR system, see http://pair-direct.uspto.gov. Should you have questions on access to the Private PAIR system, contact the Electronic Business Center (EBC) at 866-217-9197 (toll-free). If you would like assistance from a USPTO Customer Service Representative or access to the automated information system, call 800-786-9199 (IN USA OR CANADA) or 571-272-1000.

/Stephen J. Kalafut/
Primary Examiner, Art Unit 1795

Appendix

Applicant has referred to the book by R. L. Mills entitled *The Grand Unified Theory of Classical Quantum Mechanics* (Blacklight Power Inc., New Jersey, 1999; hereafter, "GUT"), which describes the existence of new energy states for the hydrogen atom that are below the conventionally accepted ground state energy. A hydrogen atom in any one of these new energy states is termed a "hydrino". According to equations (I.75a-c) on pages 19-20 of GUT, the general formula representing the energy levels for an electron with a principal quantum number, n, around the nucleus of a hydrogen atom is:

$$E_n = - (\text{Rydberg constant})/n^2 = - 13.6 \text{ electron volt}/n^2$$

Where n = 1, 2, 3, etc. and n is also = 1/2, 1/3, 1/4, etc. While the former integer values of n give energies that are conventionally understood and experimentally verified, the latter fractional values of n lead to energies of the electron in a hydrino atom which, according to Mills, represents a new "lower energy hydrogen atom".

A review of some of the main mathematical underpinnings in GUT shows that there is really no proper theoretical basis to assert the existence of the hydrino atom in view of the following discussion.

Nowhere has Mills satisfactorily established that fractional values of n arise as a natural consequence of a logical and internally consistent mathematical and scientific framework. While GUT bristles with a dense array of mathematical equations, the fractional values of n are not shown to be the unequivocal end result of Mill's theory. It appears that there is an internal break in logic in the mathematical analysis, with Mills ultimately relying on conclusory statements, such as, a nonradiative boundary condition and the relationship between the electron and a photon gives transitions in which the electron goes to a "lower" energy nonradiative state with a

smaller radius or, alternatively, that an electron can undergo a collision with an "energy hole" which allows the electron to undergo a transition to a lower energy nonradiative state with a smaller radius (pages 16-17 of GUT). In these transitions, the process involved is called a "shrinkage reaction" yielding a shrunken hydrogen atom accompanied by the release of energy. See pages 16, 17 and 114-146 of GUT.

By way of background, it is noted that there are at least two conventionally recognized approaches to the problem of obtaining the energy levels of the electron in the hydrogen atom. These are:

(a) Via a Differential Equation approach formulated as a two-point boundary value problem where boundary conditions at the nucleus and at infinity are imposed on the radial wavefunction of the electron which satisfies a second-order linear differential equation known as Schrödinger's wave equation. It is to be understood that while the complete wavefunction in spherical polar coordinates is theproduct of a radial wavefunction and angular wavefunctions, the complete wavefunction for the ground (or lowest energy) state of the hydrogen atom is independent of angular coordinates in view of the spherical symmetry of that state, and is studied only on the basis of the radial wavefunction. Thus, see attached sections 18d-18e and 21b at pages 121-124 and 139 from Pauling and Wilson's *Introduction to Quantum Mechanics* (Dover Publications, Inc., New York, 1985) and **Endnote 1**.

(b) Via an Integral Equation approach wherein the boundary conditions on the radial wavefunction of the electron are "built into" the integral equation itself rather than being imposed on it as in the differential equation formulation. In this approach, upon taking the Fourier transform of the wavefunction, subject to the boundary condition that it satisfies

Schrödinger's equation, an integral equation is obtained. Thus, see attached pages 899-900 from Morse and Feshbach's *Methods of Theoretical Physics, Part I* (McGraw-Hill Book Company, New York, 1953) and **Endnote 2**.

It is crucial to note that either approach is but a mathematical tool and that, while the integral equation approach may be mathematically more compact and, perhaps, be more convenient for solving certain problems compared to the differential equation approach, the final results given by either approach must not be mutually contradictory if a scientific theory based on these approaches is to be logical and internally consistent.

From a consideration of Mills' mathematical derivations on pages 4-5 (equations (I.5) to (I.11)), on pages 32-38, (equations (1.3) to (1.45)) and on pages 136-141 (equations (5.1) to (5.21)) of GUT, it appears that Mills' formations may be an integral equation type of approach. Specifically, the boundary condition "built into" the integral equation is an expression for the current density, and thus, the charge density of a point charge which satisfies Maxwell's equation for the electric field as given by Haus in a paper, in the *American Journal of Physics*, vol. 54, no. 12, pages 1126-1129 (1986), relating to the absence of radiation from a point charge moving at constant velocity. See page 3 of GUT. While Haus' paper is not the focus of discussion here, it is apparent that the use of a Dirac delta function $\delta(r-r_n)$, to represent the electron charge density on page 4 of GUT may be an unphysical assumption in that, whereas the electron charge density is an "observable" that is ultimately measurable, the delta function, which purports to represent it, is not, in and of itself, a function in the usual mathematical sense of the term and is physically meaningful only under an integral sign.

More specifically, it appears that Mills' integral equation approach utilizes the technique of the "Green's function". In the theory of integral equations, the Green's function is a function that satisfies a differential equation involving a Dirac delta function type of point source. A connection between the Green's function and the wavefunction is established by requiring the former to satisfy boundary conditions corresponding to those satisfied by the latter. Interpreting Mills' equations as best as one can, it is possible, though by no means certain, that Mills achieves such a connection by requiring the Green's function to satisfy boundary conditions imposed on the charge density function in Mills' equation (1.1) on page 31 of GUT. The final step in the integral equation approach is to generate an integral equation involving an integral taken over the Green's function. The solution of that equation would yield the wavefunction of the electron and, from that, leads to the energy levels of the electron in the hydrogen atom. See attached pages 808, 902 and 903 from Morse and Feshbach *op. cit.* and **Endnote 3**. It is observed that the legitimate use of a Green's function which satisfies an equation involving a Dirac delta function type of "point source" and appears, ultimately under an integral sign as the kernel of an integral equation, does not justify Mills' representation of the electron charge density, which is a "smeared out" charge distribution, as a Dirac delta function as discussed previously. Mills' lack of consistency in using properly subscripted variables as well as the absence of a logical flow in the mathematical derivations, prevents one from properly assessing the kind of approach taken in GUT.

In any event, at least some problematical issues are seen in the Mills treatment, *viz*., (i) it is not explained as to why it is physically meaningful to utilize Haus' boundary condition for a classical point charge moving in free space in order to obtain the energy levels of the electron in

a quantized system such as the hydrogen atom where the electron moves in a confined space due to its attractive coulombic interaction with the positively charged nucleus, and, (ii) there is no explanation for the catastrophic collapse of the electron into the nucleus as $n \rightarrow \infty$ in the fractional quantum number series, $1/n$, i.e., the hydrino atom implodes and ceases to exist. See pages 144-146 of GUT. The end result of Mills' integral equation approach, if such it is, fails to bear out his assertion that n must unequivocally have fractional values. In essence, it appears that the condition that n have fractional values (see equations (I.75) and (2.2) on pages 20 and 81 of GUT) is but an ad hoc statement that does not logically flow from Mills' derivation of the equation for the energy levels of the electron in the hydrogen atom and it may even represent a type of forces parameterization scheme deliberately structured to produce a desired outcome contrary to the logical flow of its mathematics or, even, common sense.

Hence, it appears that Mills' theory remains essentially unproven as discussed above and does not constitute a proper basis to demonstrate the existence, at least on theoretical grounds alone, of the so-called hydrino atom.

Furthermore, Mills' theory does not show that the conventional quantum mechanical treatment of the hydrogen atom is theoretically or experimentally flawed. Any attempt to establish a new result for the hydrogen atom that is presently unknown to quantum mechanics must cross a rather steep threshold of scientific credibility. See the attached page 2 from Bethe and Salpeter's *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977) and **Endnote 4**.

Among the many problems solved by quantum mechanics, the hydrogen atom, along with the linear harmonic oscillator and the particle-in-a-box, is one of the few scientific problems that

has received extensive theoretical and experimental treatment over many years since the first decade of the twentieth century. For a complete treatment of the hydrogen atom problem see the attachment from pages 19-1 to 19-I8 of Feynman's *Lectures in Physics,* vol. III, Quantum Mechanics (Addison-Wesley Publishing Co., Reading, Mass., 1965). The results obtained from at least one type of standard procedure for solving the radial Schrödinger equation using a power series expansion for the wavefunction of the electron inescapably lead to the conclusion that only positive integer values for n are permissible (as explained previously in Endnote 1). See attached pages 1-9 and 2-6 from Feynman *op. cit.* and **Endnote 5**. In other words, conventional theory and experiment forbid hydrino atoms.

Endnote 1

Schrödinger's wave equation for the radial wavefunction, S(ρ), is:

$$(1/\rho^2)(d/d\rho)(\rho^2 dS/d\rho) + \{-1/4 - l(l+1)/\rho^2 + \lambda/\rho\}S = 0$$

where ρ is proportional to the radial coordinate in the spherical polar coordinate system with $0 \leq \rho \leq \infty$, l us the angular momentum quantum number and λ is proportional to negative (i.e. bound) energy values. The boundary conditions are that far from the nucleus of the hydrogen atom ($\rho \rightarrow \infty$) the radial wavefunction becomes negligible, i.e.

$S \rightarrow 0$, and, at the nucleus of the atom (ρ = 0), noting that S is expressible as $e^{-\rho/2}\rho L(\rho)$ where $L=\Sigma_v a_v r^v$ is an infinite power series in ρ, substitution of the expression for S into the radial wavefunction equation results in the choice of $s = +l$ (which is a positive integer) as the only choice that will permit S to be an acceptable wavefuction, which in turn yields the boundary condition that S has a finite value at the nucleus. Note that despite the finite value of the radial

wavefunction at the nucleus, the probability of finding the electron at the nucleus, $\rho = 0$, of the hydrogen atom in its normal ground state is proportional to $4\pi\rho^2S^2$ which, of course, is zero. Upon substituting the recited expression for S into the radial wavefunction equation, recursion relations between a_v for various values of v are obtained. The recursion relations contain the principle quantum number n appearing as a multiplicative coeffience of a_v. Since S must have a proper asymptotic behavior as $\rho \rightarrow \infty$, this requires that the infinite power series be terminated after a finite number of terms which in turn, after some algebra, leads to the result that *n* must be a positive integer having values of 1, 2, 3, etc. See equations (18.29) to (18.39) and Figure 21-1 at pages 121-124 and 140 in Pauling and Wilson.

Endnote 2

Substitution of the Fourier transform of the wavefunction, $\varphi(r)$, *viz.*

$$\varphi(r) = (1/h)^{3/2} \int_{-\infty}^{\infty} j(p)e^{(2\pi i/h)pr}dp,$$

Where h is Planck's constant and p and r are momentum and spatial coordinate vectors, respectively, into the Schrödinger equation in the differential form

$$\nabla^2\varphi + (2m/h^2)\{E - V[r,(h/2\pi i)\nabla]\}\ \varphi = 0,$$

where ∇^2, E and V are the Laplacian operator, total and potential energies, respectively, followed by multiplication through by $(1/h)^{3/2}e^{(-2\pi i/h)qs}$ and an integration over r yields the desired integral equation

$$(q^2/2m)\varphi(q) + \int_{-\infty}^{\infty} \varphi(p)V(p\text{-}q,\ p)dp = E\varphi(q),\ \text{where}$$

$$V(p\text{-}q,\ p) = (1/h)^{3/2}\int_{-\infty}^{\infty}e^{(2pi/h)(p-q)s}V(r,p)dr,$$

with q being a momentum vector.

See equation (8.14) at page 900 in Morse and Feshbach.

Endnote 3

To illustrate a method of obtaining a solution for the wavefunction, φ, by the technique of Green's functions consider the Schrödinger equation for j written as:

$$[\nabla^2 + k^2]\varphi = U\varphi$$

where $k^2 = (8\pi^2 m/h^2)E$ and $U = (8\pi^2 m.h^2)V$ with E and U being the total and potential energies, respectively. A Green's function, $G_k(r|r_0)$, is introduced which satisfies

$$[\nabla^2 + k^2]G_k(r|r_0) = -4\pi\delta(r-r_0),$$

where $\delta(r-r_0)$ is a Dirac delta function representing a "point source" at r_0. The Green's function can be thought of as representing an effect at r caused by a point source r_0. The boundary conditions on $G_k(r|r_0)$ are chosen to be the same as those corresponding to the boundary conditions on the wavefunction φ. Then, by the theory of integral equations, a solution to the Schrödinger equation is:

$$\varphi(r) = -(1/4\pi) \int G_k(r|r_0)U(r_0)\varphi(r_0)dr_0.$$

See pages 808, 902 and 903 in Morse and Feshbach.

Endnote 4

Regarding the study of the hydrogen atom, note the following quotation from page 2 of Bethe and Salpeter's classic text entitled *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977):

"One of the simplest, and most completely treated, fields of application of quantum mechanics is the theory of atoms with one or two electrons. For hydrogen and the analogous ions He^{+}, Li^{++}, etc, the calculations can be performed exactly, both in Schrödinger's nonrelativistic wave mechanics and in Dirac's relativistic theory of the electron. More specifically, the calculations are exact for a single electron in a fixed Coulomb potential. Hydrogen-like atoms thus furnish an excellent way of testing the validity of quantum mechanics. For such atoms the correction terms due to the motion and structure of atomic nuclei and due to quantum electrodynamic effects are small and can be calculated with high accuracy. Since the energy levels of hydrogen and similar atoms can be investigated experimentally to an astounding degree of accuracy, some accurate tests of the validity of quantum electrodynamics are also possible."

Endnote 5

It is noteworthy that this position is also supported by a different line of argument that is independent of the solution to Schrödinger's equation. Thus, fractional values for the principal quantum number *n* would bring the electron much closer to the nucleus of the hydrogen atom than is permitted by Heisenberg's Uncertainty Principle. Feynman has presented a mathematically simple argument, in his "Lectures in Physics," vol. III, pages 2-6, to show that the size of the hydrogen atom, i.e., when *n* is 1 (rather than, say, 1/2) is perfectly consistent with the Uncertainty Principle. This argument goes as follows: from the Uncertainty Principle, if the electron is at a distance *a* from the hydrogen nucleus, then the product of its momentum and *a* must be of the order of Planck's constant. Now the total energy of the electron is the sum of its

kinetic and potential energies. Noting that the kinetic energy can be expressed in terms of the square of the momentum, upon invoking the value of the momentum from the Uncertainty Principle and minimizing the total energy in order to obtain the lowest energy level of the electron, one immediately obtains the standard result for the lowest energy level of the electron in the hydrogen atom which is consistent with *n* being 1 and no lower than 1. Since, according to Feynman, "no one has ever found (or even thought of) a way around the Uncertainty Principle…so we must assume it describes a basic characteristic of nature (page 1-9 in Feynman) it appears that Mill's fraction value for *n* is impermissible in light of the inviolability of the Uncertainty Principle.